Being aware of the choices we make

The design of a solution to a problem involves making a series of decisions, each of which may impose constraints on the subsequent stages of the design process. A decision involves making a choice from the set of alternatives available to us at each stage. Therefore, our decisions will only be as good as the alternatives we have to select from.

Being aware of the alternatives available to us has two immediate benefits :

First, if we are aware of the alternatives available to us, we are in the best position to choose the most suitable alternative for our purposes.

Second, we may sometimes find ourselves in a situation where there is no rational way to decide between the alternatives available to us. The best we can do is to make a 'blind' choice while keeping in mind that alternatives are available. Should our chosen course of action lead us to a dead end, we would at least be aware that we made a choice earlier in the design and another course is still available to us.

A careful investigation into the alternatives available at each stage is essential to the success of a disciplined design process. A failure to investigate the existence of alternatives might be the difference between an ugly solution and an elegant one. It may even result in the inability to reach the solution.

We illustrate this with the following problem:

Problem Statement. A man has a boat that moves at constant speed in still water. He starts on a boat trip, moving upstream. After 15 minutes, he realizes he dropped his hat the instant he started the trip, so he turns around to get it. When he finally catches up to the hat, he is 2 kilometers downstream from where he began the trip. Assuming the turnaround time is negligible, how fast is the stream moving relative to an observer on the bank? **(End of Problem Statement.)**

In our treatment, the unit of distance is a kilometer and the unit of a time interval is a minute.

The problem statement provides us with a time interval and a distance and requires to calculate a speed. We are thus invited to establish a relationship between these quantities and, indeed, we have the well known :

$$(0) \qquad s = \frac{d}{t}$$

where s is a speed, d is a distance and t is a time interval. We elect to investigate the use of this relation because this is the simplest relationship between the quantities of interest that we know of and there is little else to do at this stage. We thus select s to be the speed of the stream, and d the distance traveled by it in time interval t .

Focussing on (0) , we recall that it is a relation defined relative to some observer. In general, we have the freedom to choose the observer. However, our problem statement specifies that s must be computed with respect to an observer on the bank, i.e. to a stationary observer – our choice is all but made for us.

To compute s , we need an appropriate d and t . The distance travelled by the stream is equivalent to the distance travelled by the hat in a given time interval, therefore we have

(1) $d = 2$

and the time it takes to cover this d is the time taken by the round trip of the boat. i.e.

(2) $t = (tb_{up} + tb_{down})$

where tb_{up} and tb_{down} are the time intervals during which the boat travels upstream and downstream respectively.

From the problem statement, we have tb_{up} . We are done if we can find a suitable tb_{down} . Noting that we are only working with the concepts of speed, distance, and time intervals, and noting that (0) is the simplest relationship amongst them, we re-arrange (0) and substitute the appropriate quantities to arrive at :

(3) $$tb_{down} = \frac{db_{down}}{sb_{down}}$$

where db_{down} is the distance covered by the boat downstream and sb_{down} is the speed of the boat downstream.

Once again, we have to make a choice of observer, and, unlike before, this time we are not constrained in this choice. How do we choose? We observe that time interval tb_{down} closes when the boatman picks up the hat. Also, the distance db_{down} has the hat as one end point. The motion of the boat as modeled in (3) is thus defined relative to the hat, and this suggests that we make the hat the observer.

Now, the stream is stagnant relative to the hat. We know that the speed of the boat in stagnant water is constant. Therefore the speed of the boat relative to the hat is constant, whatever the direction. Additionally, the distance travelled by the boat relative to the hat is the same in both directions – whatever the distance the boat travels away from the hat, it must cover this exact distance to get back to it. Thus we have

$$sb_{up} = sb_{down}$$

and,

$$db_{up} \quad = \quad db_{down}$$

and reformulating (0) as

$$tb_{up} \quad = \quad \frac{db_{up}}{sb_{up}} \ ,$$

we can conclude

$$(4) \quad tb_{down} \quad = \quad tb_{up} \quad = \quad 15 \ .$$

Now we have all the information we need to solve (0) and hence our problem is solved. The computation of the actual answer is left as an exercise to the interested reader.

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This problem was found in the question paper of a Competitive Exam where the examinee is required to solve it within forty-five seconds. It has been observed that most people implicitly choose a stationary point for the observer all the time, thus leading to excessive algebraic manipulations which are often given up on before the solution is reached.

By a careful analysis of the options available to us, it is more likely that we will make a wise choice of observer and hence it is conceivable that this problem could be solved within the stipulated time period. This just goes to show how a disciplined analysis of the situation can go a long way in improving the effectiveness of our problem solving.

In closing, I would like to thank Jeremy Weissmann for his contribution to this document. He helped refine my original solution and also helped me identify that the crux of the solution lay in the choice of observer.

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