An embellishment of WF202

The heart of WF202 consists of a proof of:

for predicate transformer f,

$$f \text{ is punctual} \wedge f \text{ is monotonic} \\ \Rightarrow \\ f \text{ is conjunctive}.$$

During an examination with student Rob Goud we designed a proof slightly different from the one recorded in WF202. It reads as follows.

For any x, y we observe

$$
\begin{array}{ll}
 & [\, f.(x \wedge y) \equiv f.x \wedge f.y \,] \\
\equiv & \{ f \text{ is monotonic, hence} \\
 & \quad [\, f.(x \wedge y) \Rightarrow f.x \wedge f.y \,] \ \} \\
 & [\, f.x \wedge f.y \Rightarrow f.(x \wedge y) \,] \\
\equiv & \{ \text{predicate calculus} \} \\
 & [\, (f.x \Rightarrow f.(x \wedge y)) \vee (f.y \Rightarrow f.(x \wedge y) \,] \\
\Leftarrow & \{ \text{predicate calculus} \} \\
 & [\, (f.x \equiv f.(x \wedge y)) \vee (f.y \equiv f.(x \wedge y)) \,] \\
\Leftarrow & \{ f \text{ is punctual} \}
\end{array}
$$

$\Leftarrow$ { f is punctual }

$[\ (x \equiv x \wedge y) \ \vee \ (y \equiv x \wedge y)\ ]$

$\equiv$ { one or two steps of predicate calculus }

true.

* * *

The difference is that here f's punctuality and f's monotonicity are each used exactly once, contrary to WF202 where f's monotonicity is used twice. We consider this as an embellishment — no: as an improvement!

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20 June 2005