Three characterizations of extreme solutions

The strongest solution x of equation

$$y : [b.y]$$

can be characterized by either (0), (1), or (2) given by

(0a) $[b.x]$

(0b) $(\forall y : [b.y] . [x \Rightarrow y])$

(1a) $[b.x]$

(1b) $[x \equiv (\forall y : [b.y] : y)]$

(2a) $(\exists y : [b.y] : [y \Rightarrow x])$

(2b) $(\forall y : [b.y] : [x \Rightarrow y])$

The proofs that these characterizations are pairwise equivalent are elementary exercises in predicate calculus and are not given here.

Characterization (0) is the most traditional one, and (1) is traditional too. In EWD 1148, Edsger W. Dijkstra came up with (2). The only thing that I want to point out is that (1) is the formally strongest characterization of x, and (2) the formally weakest one. It might be advantageous to know this when dealing with extreme solutions.

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