A trifle on the stability of $f.x \leq g.y$

(for the record)

We consider relation

R: $f.x \leq g.y$,

in which f is an integer function not depending on y and g is an integer function not depending on x. We ask ourselves under what conditions statement

S: $y := y + Y$

maintains R. To that end we observe

$$
\begin{array}{ll}
 & wlp.S.R \\
= & \quad \{\text{axiom of assignment}\} \\
 & f.x \leq g.(y+Y) \\
\Leftarrow & \quad \{\text{using that } R \text{ is a precondition}\} \\
 & g.y \leq g.(y+Y)
\end{array}
$$

- $$
\begin{array}{ll}
\Leftarrow & \quad \{\text{assuming monotonicity of } g\} \\
 & y \leq y + Y \\
= & \quad \{\text{calc.}\} \\
 & 0 \leq Y
\end{array}
$$

- $$
\begin{array}{ll}
\Leftarrow & \quad \{\text{assuming antimonotonicity of } g\} \\
 & y + Y \leq y \\
= & \quad \{\text{calc.}\} \\
 & Y \leq 0
\end{array}
$$

As a result, increments of y maintain R for monotonic g and decrements of y maintain R for antimonotonic g. For changes in x we have similar requirements for f.

* * *

In the development of multiprograms relations of the form $f.x \leq g.y$ are very wanted because of the above stability properties and -hence- abound. I think that in an orderly treatment of multiprogramming the above theorem has to be mentioned explicitly, in spite of its very simplicity. Hence this note.

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