On the occasion of the installation of a blackboard.

When the Saddleback Search went around, one of us solved the following problem: for a matrix $M[i,j\colon 1 \le i \le X \wedge 1 \le j \le Y]$ of 0's and 1's with all X rows ascending and all Y columns descending compute p and q such that

$$p = \underline{N}(i,j\colon\ M[i,j]=0)$$
$$q = \underline{N}(i,j\colon\ M[i,j]=1)$$

With

$$p = \underline{N}(i,j\colon x<i\le X \vee y<j\le Y\colon\ M[i,j]=0)$$

as a main constituent of the invariant relation, and with

omission of further details

the program is

```
|[ x, y : int
 ; x, y := X, Y ; p, q := 0, 0
 ; do  x ≠ 0 ∧ y ≠ 0
     → if M[x,y] = 0 → p := p+y ; x := x-1
       ▯ M[x,y] = 1 → q := q+x ; y := y-1
       fi
   od
]|
```

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Recently the term "projection" has been coined.

It refers to a technique of studying programs by confining the attention to a well-chosen part of the state space.
Independently of the value of M the above program

- establishes $p+q = X\,Y$
- terminates in its repetition
- does not abort in its selection,

properties it shares with the less specific non-deterministic program

S:
```
|[ x, y : int
; x, y := X, Y ; p, q := 0, 0
  { P :  p + q + x.y = X.Y   : invariant }
; do x ≠ 0 ∧ y ≠ 0
    → if true → p := p + y ; x := x - 1
      [] true → q := q + x ; y := y - 1
      fi
  od { R : p + q = X.Y }
]|,
```

which is the result of projecting the first program on the space spanned by x, y, p, and q.

Since the partitioning of the X.Y elements of M in 0's and 1's can be chosen arbitrarily, program S is nondeterministic enough to generate any partitioning of X.Y in two summands.

When S appeared on the blackboard a nice proof that S can generate any partitioning of $X.Y$ was given by M. Rem and R.R. Hoogerwoord. The recording of that proof forms one of the reasons for writing this manuscript.

The argument is as follows.

Let A and B be such that

H: $\quad A + B = X.Y$.

Strengthen the invariant P with

Q: $\quad p \leq A \;\wedge\; q \leq B$.

Upon completion of S we conclude from $H \wedge Q \wedge R$

$$p = A \;\wedge\; q = B$$

In order to maintain Q the alternative construct of S is replaced by

$\{H \wedge P \wedge Q \wedge x \neq 0 \wedge y \neq 0\}$
<u>if</u> $p + y \leq A \rightarrow p := p + y ; \ldots$
$[\,]$ $q + x \leq B \rightarrow q := q + x ; \ldots$
<u>fi</u>

requiring a proof of the absence of abortion.

Assume: $\quad p + y > A$
H: $\quad A + B = X.Y$
P: $\quad \underline{X.Y = p + q + x.y} \;\; +$

$\quad y + B > q + x.y$

hence: $\mathcal{B} > q + (x-1)\, y$

$x \geq 1 \land y \geq 1$: $\mathcal{B} > q + (x-1)$.

Hence $\mathcal{B} \geq q + x$.

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Two of us, trying to think about kind of an inverse problem, found how to factorize a number N into two factors:

```
|[ p : int
; p, x, y := N-1, 1, 1
  { p + x.y = N  ∧ (x|p ∨ y|p)  :  invariant }
; do  p ≠ 0
   →  if  x|p → p:= p-x ; y:=y+1
      [] y|p → p:= p-y ; x:=x+1
      fi
  od
  { x.y = N }
]|.
```

The proof that this program can generate any factorization of N is left as an exercise.

*

We just wished to record what was done on the blackboard – mostly between five and half past five – shortly after its installation.
There is a whiteboard next to the blackboard. It tells us in an eye-catching arrangement of diagrams

and full-colour-photographs about computing science. In any case the advantage of a blackboard over such a whiteboard is that all nonsense on it can be erased.

Eindhoven,
24 November 1981

R.W. Bulterman,
W.H.J. Feijen,
A.J.M. van Gasteren.