(Comments on "Mathematics and Plausable Reasoning, by G. Polya ".)

[0]   Induction and Analogy in Mathematics, G. Polya .
([1]   Patterns of Plausable Inference, G. Polya .)

A problem on space partitioning.

        In [0] , G. Polya addresses the question of "in how many parts space
is divided by 5 planes in general position".
He begins by arguing that he doesn't want to clarify the notion of 'in general
position' because "the presentation should not be too technical" and because
leaving the notion somewhat hazy brings us "nearer to the mental attitude of the
naturalist" . Then a text of seven pages follows, giving an account of how a
clever naturalist might proceed when using the techniques of "generalization,
specialization and analogy" . The text ends with the firm suspicion that the
answer to the problem might be 26 , obtained by applying the principle of Poor
Man's Induction . Polya adds that "Induction renders its result probable, it
never proves them" .

        As a programmer, and as a mathematician for that matter, I know that
clarifying hazy notions in the very beginning quite often creates the possibility
of effective and efficient reasoning. So let us try what happens if we take that
attitude.
We define:

—   ' n planes in space are in general position' means that
         the intersection of each plane with the remaining planes forms
         a pattern of ' n-1 lines in plane which are in general position' ;

—   ' n lines in plane are in general position' means that
         the intersection of each line with the remaining lines forms
         a pattern of ' n-1 points in line which are in general position' ;

—   ' n points in line are in general position' means that
         all n points are distinct.

If the number of parts in which (space, plane, line) is divided by n (planes,
lines, points) in general position is denoted by ($S(n), P(n), L(n)$) respectively,
then

— $L(0) = 1$, $L(n+1) = L(n) + 1$,
         because this is obvious;

— $P(0) = 1$, $P(n+1) = P(n) + L(n)$,
         because each part of line bisects one part of plane;

— $S(0) = 1$, $S(n+1) = S(n) + P(n)$,
         because each part of plane bisects one part of space,

so that $S(5) = 26$.

This note is written (1) to oppose the naturalist's view of life to that of a
mathematician, and (2) to exercise in precise reasoning without introducing too
heavy formalism; (3) to welcome Netty .

Eindhoven, 13 February 1981 .   W.H.J. Feijen