The barber's paradox is no paradox

We consider a set V —the inhabitants of a village— and a binary relation $\triangleright$ —shaves— on V. Under the assumption

$$b \in V \ ,$$

we derive:

$$
\begin{array}{ll}
& (\forall x : x \in V : b \triangleright x \not\equiv x \triangleright x) \\
\Rightarrow & \{ \text{ instantiation } \quad x := b \ (\text{using } b \in V) \ \} \\
& b \triangleright b \not\equiv b \triangleright b \\
= & \{ \text{ propositional calculus } \} \\
& \text{false} \ .
\end{array}
$$

conclusion: $\neg(\exists b : b \in V : \ (\forall x : x \in V : b \triangleright x \not\equiv x \triangleright x))$.

So, what is the paradox?

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