A sequel to rh129 and to EWD1061

In rh129 I gave -- as supporting evidence for the use of an explicit operator for function application -- two different definitions of functions pair, fst, and snd. These definitions are:

(0) $\quad \mathrm{pair}\cdot x\cdot y\cdot s = s\cdot x\cdot y$,
$\quad \mathrm{fst}\cdot p = p\cdot K$,
$\quad \mathrm{snd}\cdot p = p\cdot C$.

(1) $\quad \mathrm{pair}\cdot x\cdot y = (\cdot y)\circ(\cdot x)$,
$\quad \mathrm{fst} = (\cdot K)$,
$\quad \mathrm{snd} = (\cdot C)$.

Here, K, C, $\circ$, and $(\cdot x)$ are defined by:

$K\cdot x\cdot y = x$,
$C\cdot x\cdot y = y$,
$(x\circ y)\cdot z = x\cdot(y\cdot z)$,
$(\cdot x)\cdot y = y\cdot x$.

Functions pair, fst, snd satisfy $\mathrm{fst}\cdot(\mathrm{pair}\cdot x\cdot y) = x$ and $\mathrm{snd}\cdot(\mathrm{pair}\cdot x\cdot y) = y$, for all x, y. We now derive two proofs of $\mathrm{fst}\cdot(\mathrm{pair}\cdot x\cdot y) = x$ in two ways, using definition (1) and definition (0) respectively. Notice that we tacitly use that function application is left-binding, i.e. $x\cdot y\cdot z = (x\cdot y)\cdot z$.

$\mathrm{fst}\cdot(\mathrm{pair}\cdot x\cdot y)$
= { definition (1) of fst }
$(\cdot K)\cdot(\mathrm{pair}\cdot x\cdot y)$
= { definition of $(\cdot x)$ with $x,y \leftarrow K, \mathrm{pair}\cdot x\cdot y$ }
$\mathrm{pair}\cdot x\cdot y\cdot K$
= { definition (1) of pair }
$((\cdot y)\circ(\cdot x))\cdot K$
= { definition of $\circ$ with $x,y,z \leftarrow (\cdot y),(\cdot x),K$ }

(·y)·((·x)·K)

= { definition of (·x) with x,y ← y , (·x)·K }

(·x)·K·y

= { definition of (·x) with x,y ← x,K }

K·x·y

= { definition of K }

x .

This derivation consists of 7 steps, the order of which is not completely fixed; in the above derivation we followed the (not so bad) strategy to replace the left-most subexpression to which one of the rules could be applied. The symbol count -- counting each multi-letter identifier as one symbol -- for this proof yields 9,12,7,13,13,8,5,1 (sum 68). The second proof runs as follows:

fst·(pair·x·y)

= { definition (0) of fst with p ← pair·x·y }

pair·x·y·K

= { definition (0) of pair with x,y,s ← x,y,K }

K·x·y

= { definition of K }

x .

This proof consists of 3 steps; its construction leaves us no freedom. Moreover, each of the definitions of the functions involved is used once, and that is all. The symbol count for this proof is 9,7,5,1 (sum 22). Notice that each formula from the second proof also occurs in the first proof.

So much for a little story that provides some more evidence for the point made in EWD1061, namely that equational reasoning with function applications may be expected to be more efficient than reasoning with functions themselves.

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reference

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