A sequel to RH79

The way in which I solved the problem posed in RH79 is unnecessarily complex. If we take the following recurrence relation as our starting point a simpler program is possible:

$$\begin{array}{lll} & s(0) & = 0 \text{ , and :} \\ \text{for } k : k \geq 0 : & s(k+1) & = A * s(k) + 1 . \end{array}$$

This type of recurrence relation is called "linear" and "non-homogeneous". It can be transformed into a homogeneous one by introduction of an additional coordinate which, because its value is constant, can remain anonymous:

$$\begin{array}{ll} & s(0) = 0 \text{ , and :} \\ \text{for } k : k \geq 0 : & \begin{pmatrix} s(k+1) \\ 1 \end{pmatrix} = \begin{pmatrix} A & 1 \\ 0 & 1 \end{pmatrix} * \begin{pmatrix} s(k) \\ 1 \end{pmatrix} . \end{array}$$

From this relation the derivation of the following program is quite standard:

<u>invariant</u> : $\begin{pmatrix} s(N) \\ 1 \end{pmatrix} = \begin{pmatrix} a & b \\ 0 & 1 \end{pmatrix}^n * \begin{pmatrix} x \\ 1 \end{pmatrix} \wedge 0 \leq n \leq N .$

<u>program</u> :

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   n,a,b,x := N, A, 1, 0
 ; do n≠0 → do n mod 2 = 0 → n,a,b := n/2, a*a, a*b+b
            od
          ; n,x := n-1, a*x+b
   od
   { s(N) = x }.
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Eindhoven, 1985.12.12
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