The context of subderivations

In *Doing High School Mathematics Carefully* Ralph-Johan Back and Joakim Wright discuss the use of context in calculations. In the section 'Focusing and local assumptions' they describe how subderivations can make use of the context in which subterms occur. They do so through inference rules, but thanks to Jeremy Weissmann, we can express the same rules algebraically.

Focusing on a disjunct.

Suppose our term is $x \vee y$ and we wish to focus on subterm y. Then we can add to the subderivation context $x \not\equiv true$ or simply $\neg x$:

$x \vee y$

$\equiv$ {focus on y}

- |[$\neg x$

  $\triangleright$ y

  $\equiv$ $\{\ldots\}$

  $\ldots$

  ]|

Focusing on a conjunct.

Suppose our term is $x \wedge y$ and we wish to focus on subterm y. Then we can add to the subderivation context $x \not\equiv false$ or simply x:

$x \wedge y$

$\equiv$ {focus on y}

- |[x

  $\triangleright$ y

  $\equiv$ $\{\ldots\}$

  $\ldots$

  ]|

Focusing on a consequent.

Suppose our term is $x \Rightarrow y$ and we wish to focus on subterm y. Then we can add to the subderivation context $x \not\equiv false$ or simply x:

$x \Rightarrow y$

$\equiv$ {focus on y}

$\bullet$ |[x

$\triangleright$ y

$\equiv$ {...}

...

]|

Note that in the fragments above we could substitute for $\equiv$ any of $\Rightarrow$, $\Leftarrow$. Back and Wright only provide the three contexts above, but thanks to JAW74 we can add a few more. For boolean x and y we have

$x \Leftarrow y$

$\equiv$ {focus on y}

$\bullet$ |[$\neg x$

$\triangleright$ y

$\equiv$ {...}

...

]|

If x and y are real then we have

$x < y$

$\rightarrow$ {focus on y}

$\bullet$ |[$x \neq \infty$

$\triangleright$ y

$\rightarrow$ {...}

...

]|

$x \leq y$

$\rightarrow$ {focus on y}

$\bullet$ |[$x \neq -\infty$

$\triangleright$ y

$\rightarrow$ {...}

...

]|

and for natural x and y and $\rightarrow$ is any one of $=$, $\leq$, $\geq$, we have

$x * y$

$\rightarrow$ {focus on y}

$\bullet \ |[\ x \neq 0$

$\triangleright \quad y$

$\rightarrow \quad \{\ldots\}$

$\ldots$

$]|$

Birmingham, 12 October 2006