The Σ calculus Part 3.

Associativity and symmetry reveal themselves again in another postulate - referred to by 'interchange of quantifications' - that for any f

(6) $\langle \Sigma x :: \langle \Sigma y :: f.x.y \rangle\rangle = \langle \Sigma y :: \langle \Sigma x :: f.x.y \rangle\rangle$

which invites to observe for any r, s, f

$$\begin{array}{ll}
& \langle \Sigma x : r.x : \langle \Sigma y : s.y : f.x.y \rangle\rangle \\
= & \quad \{\text{trading}\} \\
& \langle \Sigma x :: [r.x] * \langle \Sigma y :: [s.x] * f.x.y \rangle\rangle \\
= & \quad \{* \text{ over } \Sigma\} \\
& \langle \Sigma x :: \langle \Sigma y :: [r.x] * [s.y] * f.x.y \rangle\rangle \\
= & \quad \{\text{interchange of quantifications}\} \\
& \langle \Sigma y :: \langle \Sigma x :: [r.x] * [s.y] * f.x.y \rangle\rangle \\
= & \quad \{* \text{ over } \Sigma\} \\
& \langle \Sigma y :: [s.y] * \langle \Sigma x :: [r.x] * f.x.y \rangle\rangle \\
= & \quad \{\text{trading}\} \\
& \langle \Sigma y : s.y : \langle \Sigma x : r.x : f.x.y \rangle\rangle
\end{array}$$

which leaves us with the more general

(7) $\langle \Sigma x : r.x : \langle \Sigma y : s.y : f.x.y \rangle\rangle = \langle \Sigma y : s.y : \langle \Sigma x : r.x : f.x.y \rangle\rangle$

which we shall also refer to as 'interchange of quantifications'. What makes (7) nice is the observation that each dummy carries with it is own range, so there is no confusion should the ranges be omitted.

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