

## The $\Sigma$ calculus Part 2.

Formulae (3) and (4) relate quantifications with the same range. (4) has an analogue for different ranges:

$$(5) \quad \langle \Sigma x : r.x \quad \wedge \quad s.x : f.x \rangle + \langle \Sigma x : r.x \quad \vee \quad s.x : f.x \rangle = \langle \Sigma x : r.x : f.x \rangle + \langle \Sigma x : s.x : f.x \rangle$$

*Proof*

$$\begin{aligned} & \langle \Sigma x : r.x \quad \wedge \quad s.x : f.x \rangle + \langle \Sigma x : r.x \quad \vee \quad s.x : f.x \rangle \\ = & \quad \{\text{trading}\} \\ & \langle \Sigma x :: [r.x \quad \wedge \quad s.x] * f.x \rangle + \langle \Sigma x :: [r.x \quad \vee \quad s.x] * f.x \rangle \\ = & \quad \{\Sigma \text{ over } +\} \\ & \langle \Sigma x :: [r.x \quad \wedge \quad s.x] * f.x \quad + \quad [r.x \quad \vee \quad s.x] * f.x \rangle \\ = & \quad \{ * \text{ over } + \} \\ & \langle \Sigma x :: ([r.x \quad \wedge \quad s.x] + [r.x \quad \vee \quad s.x]) * f.x \rangle \\ = & \quad \{[X] + [Y] \quad = \quad [X \wedge Y] + [X \vee Y]\} \\ & \langle \Sigma x :: ([r.x] + [s.x]) * f.x \rangle \\ = & \quad \{ * \text{ over } + \} \\ & \langle \Sigma x :: [r.x] * f.x \quad + \quad [s.x] * f.x \rangle \\ = & \quad \{\Sigma \text{ over } +\} \\ & \langle \Sigma x :: [r.x] * f.x \rangle + \langle \Sigma x :: [s.x] * f.x \rangle \\ = & \quad \{\text{trading}\} \\ & \langle \Sigma x : [r.x] : f.x \rangle + \langle \Sigma x : [s.x] : f.x \rangle \end{aligned}$$

*End of Proof.*

*Continued...*

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