The Σ calculus Part 1.

In analogy to addition's symmetry and associativity, we postulate that summation distributes over addition i.e. for any f and g

(3) $\langle \Sigma x :: f.x + g.x \rangle = \langle \Sigma x :: f.x \rangle + \langle \Sigma x :: g.x \rangle$

and so we are invited to observe for any r, f, g

$$
\begin{array}{ll}
 & \langle \Sigma x : r.x : f.x \rangle + \langle \Sigma x : r.x : g.x \rangle \\
= & \quad \{\text{trading}\} \\
 & \langle \Sigma x :: [r.x] * f.x \rangle + \langle \Sigma x :: [r.x] * g.x \rangle \\
= & \quad \{\Sigma \text{ over } +\} \\
 & \langle \Sigma x :: [r.x] * f.x \quad + \quad [r.x] * g.x \rangle \\
= & \quad \{* \text{ over } +\} \\
 & \langle \Sigma x :: [r.x] * (f.x + g.x) \rangle \\
= & \quad \{\text{trading}\} \\
 & \langle \Sigma x : r.x : f.x + g.x \rangle
\end{array}
$$

Hence we have

(4) $\langle \Sigma x : r.x : f.x + g.x \rangle = \langle \Sigma x : r.x : f.x \rangle + \langle \Sigma x : r.x : g.x \rangle$

To be continued...

E. Emmanuel Macaulay
9 August 2006