3.

The following proof by induction seems correct, but for some reason the equation for $n = 6$ gives $\frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \frac{1}{30} = \frac{5}{6}$ on the left-hand side, and $\frac{3}{2} - \frac{1}{6} = \frac{4}{3}$ on the right-hand side. Can you find a mistake?

"**Theorem.**

$$\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + ... + \frac{1}{(n-1) \times n} = \frac{3}{2} - \frac{1}{n}.$$

Proof. We use induction on n. For $n = 1$, clearly $3/2 - 1/n = 1/(1 \times 2)$; and, assuming that the theorem is truefor n,

$$\frac{1}{1 \times 2} + ... + \frac{1}{(n-1) \times n} + \frac{1}{n \times (n+1)} = \frac{3}{2} - \frac{1}{n} + \frac{1}{n(n+1)} = \frac{3}{2} - \frac{1}{n} + (\frac{1}{n} - \frac{1}{n+1}) = \frac{3}{2} - \frac{1}{n+1}.$$

* * *

I have always been dissatisfied with the ... notation it is at best ambiguous, at worst misleading. This question is a case in point. To see the problem in the proof we will rewrite the demonstrandum as

$$\langle \Sigma i : 1 \leq i < n+1 : 1/((i-1) * i)\rangle = 3/2 - 1/n$$

We observe

$$\begin{array}{ll} & \langle \Sigma i : 1 \leq i < n+1 : 1/((i-1) * i)\rangle \\ = & \quad \{n := 1\} \\ & \langle \Sigma i : 1 \leq i < 2 : 1/((i-1) * i)\rangle \\ = & \quad \{\text{arithmetic}\} \\ & \langle \Sigma i : 1 \leq i \wedge i \leq 1 : 1/((i-1) * i)\rangle \\ = & \quad \{\text{antisymmetry, one-point rule}\} \\ & 1/(0 * 1) \end{array}$$

But $1/0$ is undefined and so the proof of the base case is invalid.

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