2.

"There must be something wrong with the following proof. What is it?

Theorem. Let a be any positive number. For all positive integers n we have $a^{n-1} = 1$. *Proof.* If $n = 1$, $a^{n-1} = a^{1-1} = a^0 = 1$. And by induction, assuming that the theorem is true for $1, 2, ..., n$, we have

$$a^{(n+1)-1} = a^n = \frac{a^{n-1} * a^{n-1}}{a^{(n-1)-1}} = \frac{1 * 1}{1} = 1$$

so the theorem is true for $n + 1$ as well."

Well, first of all the base case should really be $n = 0$:

a^{n-1}

$=$ $\{n = 0\}$

a^{-1}

which differs from 1 when a does. We also observe

a^{n-1}

$=$ $\{n = 2\}$

a^{1}

which also differs from 1 when a does. This counterexample suffices to disprove the theorem.

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