We are asked to show

(0) $F_n \geq \Phi^{n-2}$

for $n \geq 1$. We shall prove this by induction.

Base case. $F_1 \geq \Phi^{-1}$

$F_1 \geq \Phi^{1-2}$
$=$ {definitions of F_n and Φ^n}
$1 \geq 2/(1+\sqrt{5})$
$=$ {arithmetic}
$2 \geq 1+\sqrt{5}$
$=$ {arithmetic}
true

Base case. $F_2 \geq \Phi^0$

$F_2 \geq \Phi^0$
$=$ {definitions of F_n and Φ^n}
$1 \geq 1$
$=$ {reflexivity of $\geq$}
true

Inductive case. $F_{n+1} \geq \Phi^{n-1}$

F_{n+1}
$=$ {definition of F_n}
$F_n + F_{n-1}$
$\geq$ {inductive hypothesis}
$\Phi^{n-2} + \Phi^{n-3}$
$=$ {bearing in mind the demonstrandum, we factor out Φ^{n-1}}
$\Phi^{n-1} * (\Phi^{-1} + \Phi^{-2})$
$=$ {see below}
Φ^{n-1}

We can show that $\Phi^{-1} + \Phi^{-2} = 1$ as follows:

$\Phi^{-1} + \Phi^{-2}$
$=$ {exponents and division}
$1/\Phi + 1/\Phi^2$
$=$ {common denominator}
$(\Phi + 1)/\Phi^2$
$=$ {$1 + \Phi = \Phi^2$}
Φ^2/Φ^2
$=$ {properties of /}
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