(1.1, 1) Multiple assignment

Knuth asks us to show the values of four variables, a, b, c and d can be rearranged by a sequence of replacements. More precisely, he asks us to solve:

$\{a = A \quad \wedge \quad b = B \quad \wedge \quad c = C \quad \wedge \quad d = D\}$

rearrange

$\{a = B \quad \wedge \quad b = C \quad \wedge \quad c = D \quad \wedge \quad d = A\}$

This problem could be tricky, since we have ensure for example, that a gets its new value from b before b is changed, but d must get a's original value before a is changed. Restricting ourselves to Knuth's assignment, we have to pay attention to the order in which the variables are updated and introduce an auxiliary variable.

We can avoid all this by using the multiple assignment command.

$\{a = A \quad \wedge \quad b = B \quad \wedge \quad c = C \quad \wedge \quad d = D\}$

$a, b, c, d := b, c, d, a$

$\{a = B \quad \wedge \quad b = C \quad \wedge \quad c = D \quad \wedge \quad d = A\}$

Proof

$$
\begin{array}{ll}
 & (a = B \quad \wedge \quad b = C \quad \wedge \quad c = D \quad \wedge \quad d = A)(a, b, c, d := b, c, d, a) \\
= & \quad \{\text{substitution}\} \\
 & b = B \quad \wedge \quad c = C \quad \wedge \quad d = D \quad \wedge \quad a = A
\end{array}
$$

Simple.