1. Inspired by subtraction. To recap we must prove

(0) $x * y * z \quad = \quad 1$

given

(1) $m \quad = \quad a^x$

(2) $n \quad = \quad a^y$

(3) $a^2 \quad = \quad (m^y \quad * \quad n^x)^z$

where all variables are of type integer.

2. We will write x^y as $x \leftarrow y$ for any x and y. Thus our givens are now

(1a) $m \quad = \quad a \leftarrow x$

(2a) $n \quad = \quad a \leftarrow y$

(3a) $a \leftarrow 2 \quad = \quad (m \leftarrow y \quad * \quad n \leftarrow x) \leftarrow z$

3. $\leftarrow$ has a higher binding power than $*$ and enjoys the following properties:

(4) $(x * y) \leftarrow n \quad = \quad x \leftarrow m \quad * \quad y \leftarrow n$

(5) $x \leftarrow (m + n) \quad = \quad x \leftarrow m \quad * \quad x \leftarrow n$

(6) $(x \leftarrow m) \leftarrow n \quad = \quad x \leftarrow (m * n)$

(7) $m = n \quad \equiv \quad a \leftarrow m \quad = \quad a \leftarrow n$

We refer to to (4) with the catchphrase 'right distributivity', (5) with 'left distributivity' and (6) with 'trading'.

4. We calculate

$$
\begin{array}{ll}
 & a \leftarrow 2 \\
\equiv & \quad \{(3a)\} \\
 & (m \leftarrow y \quad * \quad n \leftarrow x) \leftarrow z \\
\equiv & \quad \{\text{right distributivity}\} \\
 & (m \leftarrow y) \leftarrow z \quad * \quad n \leftarrow x \leftarrow z \\
\equiv & \quad \{\text{trading}\} \\
 & m \leftarrow (y * z) \quad * \quad n \leftarrow (x * z) \\
\equiv & \quad \{(1a), (2a)\} \\
 & (a \leftarrow x) \leftarrow (y * z) \quad * \quad (a \leftarrow y) \leftarrow (x * z) \\
\equiv & \quad \{\text{trading}\} \\
 & a \leftarrow (x * y * z) \quad * \quad a \leftarrow (y * x * z) \\
\equiv & \quad \{\text{right distributivity}\} \\
 & a \leftarrow (2 * x * y * z)
\end{array}
$$

The rest is easy:

$$\begin{array}{ll} & a \leftarrow 2 \quad = \quad a \leftarrow (2 * x * y * z) \\ \equiv & \{(7), \text{arithmetic}\} \\ & 1 \quad = \quad x * y * z \end{array}$$

5. This is a definite improvement over EEM9. It would simply not have been possible without Jeremy Weissmann's papers on subtraction. Hopefully he will publish them as JAWs soon.

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EEM9A

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