

Portia returns

Our friend Portia is back with another problem for us. This time she puts her picture into one of *three* caskets and places the following inscriptions on them:

Gold casket: The portrait is in here.

Silver casket: The portrait is in here.

Lead casket: At least two of the caskets have a false inscription.

Which casket should the suitor choose?

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A very quick introduction to conjunction and negation

A *conjunction* is a proposition of the form ‘ X and Y ’. We express such propositions mathematically as $X \wedge Y$.

Last time we met the formula $X \equiv \perp$, which we can interpret as ‘ X is false’. Here we meet an equivalent way of writing it:

$$\begin{aligned} X &\equiv \perp \\ &\equiv \{ \text{negation} \} \\ &\quad \neg X \end{aligned}$$

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Back to Portia’s challenge. We express all we know about the problem mathematically as follows:

- $G \equiv$ ‘The portrait is in the gold casket.’
- $S \equiv$ ‘The portrait is in the silver casket.’
- $L \equiv$ ‘The portrait is in the lead casket.’
- $G_i \equiv$ ‘The inscription on the gold casket.’

- $Si \equiv$ 'The inscription on the silver casket.'
- $Li \equiv$ 'At least two of Gi , Si , Li are false.'
- $Gi \equiv G$
- $Si \equiv S$
- Exactly one of G , S and L is true.

Now Li is either true or false. Let us assume that Li is false. Then we have

$$\begin{aligned}
 & \neg Li \\
 \equiv & \quad \{ \neg Li \equiv \text{'less than two of } Gi, Si, Li \text{ are false'} \} \\
 & Gi \wedge Si \\
 \equiv & \quad \{ Gi \equiv G; Si \equiv S \} \\
 & G \wedge S \\
 \equiv & \quad \{ \text{Exactly one of } G, S \text{ and } L \text{ is true} \} \\
 & \perp
 \end{aligned}$$

So $Li \equiv \top$. We calculate once more

$$\begin{aligned}
 & Li \\
 \equiv & \quad \{ Li \equiv \text{'at least two of } Gi, Si, Li \text{ are false'} \} \\
 & \neg Gi \wedge \neg Si \\
 \equiv & \quad \{ Gi \equiv G; Si \equiv S \} \\
 & \neg G \wedge \neg S \\
 \equiv & \quad \{ \text{Exactly one of } G, S \text{ and } L \text{ is true} \} \\
 & L
 \end{aligned}$$

So the portrait is in the lead casket.

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